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Mohammad Abdul Azim
Zeyar Aung
Weidong Michael Xiao
Vinod Khadkikar

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Data & Network Analytics Research Group (DNA)
Computing and Information Science Program,
Masdar Institute of Science and Technology,
PO Box 54224, Abu Dhabi, UAE.

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Mohammad Abdul Azim, Zeyar Aung, Weidong Michael Xiao and Vinod Khadkikar
Masdar Institute of Science and Technology
Abu Dhabi, United Arab Emirates
{mazim, zaung, mwxiao, vkhadkikar}@masdar.ac.ae

***Abstract*—Wireless sensor network localization technique remains an open research topic due to its challenges on reducing the location estimation error and cost of the localization algorithm itself. For a large mobile network localization cost becomes increasingly important due to the exponential increment of the algorithmic cost. Conversely sacrificing localization accuracy to a great extent is not acceptable at all. To address the localization problem of wireless sensor network this paper presents a novel algorithm based on cross-entropy (CE) method. The proposed centralized algorithm estimates location information of the nodes based on the measured distances of the neighboring nodes. The algorithm minimizes the estimated location error by using the CE method. Simulation results compare the proposed CE approach with DV-Hop and Simulated Annealing-based localizations and show that this approach provides a balance between the accuracy and cost. When compared with DV-Hop, the CE approach is costlier but much more accurate. When compared with Simulated Annealing-based method, this approach offers the same level of accuracy but is significantly less costly.**

I. INTRODUCTION

Sensor network node location information is important for numerous reasons. In many cases the sensed data has no value without the location information. The location information can be used by routing and other protocols, algorithms and services. The straightforward solution to the localization problem of equipping nodes with GPS receivers is not a suitable option because GPS receivers require line of sight to GPS satellites. Moreover GPS is costly and power hungry. Therefore for the randomly deployed sensor networks various localization algorithms has been introduced where only a small number of sensor nodes are equipped with GPS receivers and other sensor nodes derive their locations by using the localizations techniques [1][2].

Though localization is not a recent topic it still has issues and challenges to handle because some solutions are not cheap and some have unexpected level of errors. WSN Localization techniques are largely categorized into range-based and range-free localizations. The range-based technique involves in deriving absolute distances or angles whereas the range-free technique involves in deriving distances from non-anchor nodes to anchor nodes. Well known range-based localization techniques are receive signal strength indicator (RSSI) [3], angle-of-arrival (AoA), time of arrival (ToA) [4][5] or time difference of arrival (TDoA) [6][7] etc.

Ideally distance can be measured from transmit and receive signal strengths of radios. If transmit and receive signal strengths are p_i and p_j than the distance can be measured as $d_{ij} = \sqrt[\beta]{p_i/p_j}$. Where β is known as path loss exponent and can be calculated by measuring power at unit distance. But this ideal situation never exists because of the presence of noise. Ref [8] describes the source of noise that can affect the localization estimation from signal strength. Practically, RSS estimation is affected by log-normal shadowing [9]. Where the receive signal varies as $[\mu, \sigma^2]$. Where μ and σ are mean and variance and often taken as zero and one respectively. The error in signal strength estimation introduces error in measured distances. Therefore RSSI algorithm has low accuracy primarily because of multi-path fading. One straight forward solution is taking average (such as auto regressive moving average (ARMA) [10]) of power measurements before calculating the location of the nodes. Unfortunately this approach requires a large number of measurements to get a desired result [11][12][13]. Taking the measurement requires active transmission therefore costly in terms of energy usage.

In time-based methods like ToA and TDoA propagation time is used to derive the distance. But these time-based protocols suffers where the line of sight does not exist. AoA is highly accurate but requires expensive hardware. Due to the specific hardware requirement for the range-based approach a range free approach is considered more appropriate in the context of WSNs to limit the hardware cost of the nodes.

Centroid scheme [14] and DV-Hop scheme [15] are well known range-free schemes in the literature. In centroid scheme anchors broadcast their locations. Nodes receive the broadcasts and calculate node position by a simple measure of centroid by $(x_{est}, y_{est}) = (\sum x_i/N, \sum y_i/N)$. Here (x_i, y_i) is the coordinate of i_{th} anchor and N is the total number of anchors where the node is receiving beacons. This coarse grain localization algorithm is simple, lightweight and easy to implement. A number of weighted centroid localization is proposed to improve the accuracy by incorporating weights for each neighbor nodes [16][17]. Further improvement of the scheme is made by incorporating the adaptive weight for the centroid algorithm [18].

Well referred DV-Hop algorithm [15] is based on distance vector routing. Nodes calculate the hop distances from the anchors. Then the distance is measured by multiplying the hop distance to the average hop size. Where the hop size of the anchor is calculated by $Hopsize_i = \sum \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} / \sum(h_j)$. Here

(x_i, y_i) and (x_j, y_j) are the coordinates of anchor i and j and h_j is the hop distance from anchor j to i. In RSSI based DV-Hop (RDV-Hop) the authors combine and develop a hybrid localization algorithm that incorporates RSSI and DV-Hop schemes and found improvement over DV-Hop accuracy [19].

Ref [20] uses maximum likelihood (ML) estimation technique to estimate the position of the node by minimizing the difference between the measured and estimated distances. ML uses well-known minimum mean square error (MMSE) [22] algorithm for this estimation. ML suffers from poor accuracy if the number of neighbors is small [12][21]. Simulated Annealing (SA)-based localization [23] provides similar minimization technique where the minimization is performed by the optimization algorithm known as simulation annealing. But this scheme requires a large computational resource to solve the optimization problem.

Therefore among the localization algorithms in state of the art some are costly in terms of hardware, some are costly in terms of energy and computation, and some are simply too inaccurate to be practically used. Attempt to get a reasonable solution we formulate a localization algorithm that uses cross-entropy (CE)-based optimization technique [24] while deriving the x, y coordinates of the non-anchor sensor nodes in the network.

The rest of the paper is organized as follows: Section II discusses the proposed CE method of localization that comprises of distance measurement and collection steps, definition of cost function in our case along with CE optimization technique. Section III presents the simulation results to justify the necessity of such proposal and finally Section IV concludes the paper along with future directions.

II. CROSS-ENTROPY ALGORITHM FOR LOCALIZATION

Primarily we have N number of nodes randomly deployed in the network, among them A number of nodes are anchor nodes. The localization algorithm needs to determine x and y coordinates of $N-A$ number of nodes. CE-based localization technique is location estimation technique where the location is estimated based on the derived distances of the nodes from its neighborhood. The distance is calculated based on transmit-receive signal strengths measures. Fig. 1 shows the steps in detail for the proposed CE-based localization algorithm.

A. Collecting measurements

During the initialization of the protocol each node in the network:

- Creates a neighbor list
- Measures neighbor distances by transmit-receive signal strengths
- Updates central computer with aforementioned information via sink

Upon receiving data the central computer uses CE-based localization algorithm and derive the unknown locations for the non-anchor nodes. Before going into the CE method we first define the cost function used by the optimization algorithm.

B. Cost function

Due to the unreliable nature of the wireless medium the distance measure introduces error. A common approach is to estimate the location of the node by minimizing the estimated error [20][23]. The CE method incorporate the same cost function to be minimized. Let d_{ij} is the measured distance among node i and j. Let (x_i, y_i) and (x_j, y_j) is the estimated coordinates of the node i and j by the algorithm. Here the estimated distance is $\hat{d_{ij}} = \sqrt{(\hat{x}_i - \hat{x}_j) + (\hat{y}_i - \hat{y}_j)}$. Therefore the cost function to be minimized can be expressed as

$$cost_i = \sum_{i=A+1}^{N} \sum_{j \epsilon n_i} (\hat{d}_{ij} - d_{ij})^2 \tag{1}$$

Where n_i is the set of all neighboring nodes of node i. With the measured distances and the aforementioned cost function CE algorithm solve the localization problem in an iterative learning manner.

C. Cross-entropy optimization algorithm

CE localization algorithm attempts to find the best coordinate of the unknown sensor node by minimizing the estimated error. The underlying technique in CE optimization is to generate samples based on the means and variances. Algorithm than selects the best sample as next state while it learns about the next generation sample means and variances based on the best set of samples in the population. The CE algorithm first generates random states for all nodes. It then generates a set of populations for each state based on the means and variances. The initial means and variances can simply be random numbers. Algorithm then finds the cost for all the population based on the cost function. If the minimum cost of the population set is less than the cost function of the current state than the state is updated otherwise a new set of population is generated. In each update of state the algorithm learns about a better sample generation characteristics. Where the characteristics can be defined as the mean and variance used to generate the samples. Therefore if there is an instance of updated state the mean and variance is also updated based on the best population set. CE algorithm updates the states iteratively until the cost or error is within the acceptance limit.

For each unknown node n_i the localization algorithm first randomly generates the coordinates (x_i, y_i) alternatively known as states of the nodes where n_i is a set of all non-anchor nodes denoted by $n_1 : n_{N-A}$. Algorithm also initializes means μ and variances σ for all x_i and y_i. Generally the initial means and variances are set of random numbers and set of ones respectively. The cost of all the initialized states of the nodes are determined and subsequently known as initial $BestCost_i$.

After initialization CE algorithm enters into an iterative mode and update the states until the desired refinement is achieved. This control parameter is known as variance minimum γ. Another important control parameter is the learning rate. Generally two different learning rates are used for the means and variances denoted as α and β respectively.

The iterative method starts with generating a population of S number of samples for all x_i and y_i based on the means and variables of corresponding x_i and y_i. The samples are than evaluated and rated by the cost of a particular sample. If the cost of the best sample is less than $BestCost_i$ than the $BestCost_i$ is replaced by the cost of the best sample. The state (x_i, y_i) is subsequently updated with the best sample for the particular node.

Another parameter of the algorithm is update sample number M. Algorithm then select the best M samples and find the mean and variance of the samples by $x\mu best_i = mean(xbest_1 : xbest_M)$ and $x\sigma best_i = std(xbest_1 : xbest_M)$ respectively. The mean of the best samples are used to update the corresponding mean by x_i by $x\mu_i = \alpha * x\mu_i + (1-\alpha) * x\mu best_i$ for the next generation of samples. Similarly $x\sigma_i = \beta * x\sigma best_i + (1-\beta) * x\sigma_i$ is used to update the variances of x_i. $y\mu_i$ and $y\sigma_i$ are updated in a similar fashion. The trained means and variances are the key properties of the next generation of samples. Superior samples in successive generations help the algorithm estimating better states (coordinate in our case) in successive iterations.

Alternatively if the cost of the best sample is less than $BestCost_i$ than the population set is discarded and another set of samples is generated. After completion of iterations the final state of i is the estimated location of the particular sensor node.

III. SIMULATION RESULTS

We simulate the CE-based localization algorithm in Matlab. A total number of 100 nodes are placed in 100m×100m field. Here four anchor nodes are placed in the four corners of the field and rests of the nodes are placed randomly in the whole area. We assume that the network is equipped with radios having uniform transmission range denoted by R. Here radio range R is taken as 20m. We simulate error in distance measurement with log-normal shadowing effect [9] described in Section I with mean μ and variance σ as 0 and 1 respectively.

CE control parameter variance minimum γ needs to be small enough to run the simulation reasonably long enough to get a good estimation. Then again setting γ too small make the simulation slow without much improvement. We set $\gamma = 10^{-3}$ in our case. Learning rates α and β are set as 0.7 and 0.9 respectively. Finally sample number S and best sample number M are taken as 100 and 50 respectively.

Fig. 2 shows the sensor field with normalize distances where the anchors are 45% of the total nodes. In this specific arrangement the error is very small. In our results we present two different types of errors: (i) error in each node defined as normalize distance between the original and estimated node coordinates and (ii) average error in the field defined in Equation 2 [23].

$$error = (1/(N-A)) * (\sum_{i=A+1}^{N} ((x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2)/R^2) \tag{2}$$

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Cross-entropy based localization algorithm
  $N$: Total nodes
  $A$: Anchor nodes
  $\mu$: Means
  $\sigma$: Variances
  $\alpha$: Learning rate for means
  $\beta$: Learning rate for variances
  $\gamma$: Variance minimum
Node level measurements for all node $i$
  Create neighbor list
  Measure distances by Tx-Rx signal strengths
  Update central computer via sink
Algorithm level at central location
  for all unknown node $i$
      Randomly initialize $(x_i, y_i)$ coordinates
      Randomly initialize $\mu$ and $\sigma$ for $x_i$ and $y_i$
      Find cost for $(x_i, y_i)$ and assign to initial $BestCost_i$ by
          $\sum_{i=M+1}^{N} \sum_{j \epsilon N_i} (\hat{d}_{ij} - d_{ij})^2$
  end
  while $(max(\sigma) < \gamma)$
      for all $i$
          Generate $S$ samples for $x_i$ and $y_i$
          Find costs for corresponding samples
          if (min cost of the samples $< BestCost_i$)
            Update state $(x_i, y_i)$ with the best sample
            Update $BestCost_i$
            Update $\mu$ and $\sigma$
            Select $M$ number of best population
              $(xbest_1, ybest_1) \ldots (xbest_M, ybest_M)$
            Take $\mu best$ and $\sigma best$ of the selected bests
              $x\mu best_i = mean(xbest_1 : xbest_M)$
              $y\mu best_i = mean(ybest_1 : ybest_M)$
              $x\sigma best_i = std(xbest_1 : xbest_M)$
              $y\sigma best_i = std(ybest_1 : ybest_M)$
            Update $\mu$ and $\sigma$ with $\alpha$ and $\beta$ respectively
              $x\mu_i = \alpha * x\mu_i + (1-\alpha) * x\mu best_i$
              $y\mu_i = \alpha * y\mu_i + (1-\alpha) * y\mu best_i$
              $x\sigma_i = \beta * x\sigma best_i + (1-\beta) * x\sigma_i$
              $y\sigma_i = \beta * y\sigma best_i + (1-\beta) * y\sigma_i$
          end
      end
  end
```

Fig. 1. Cross-entropy based localization algorithm

Where, (x_i, y_i) and $(\hat{x}_i, \hat{y}_i)$ are the absolute and estimated locations of the node i. N and A are total number of nodes and total number of anchors in the network [23].

One common downside of the cost minimization techniques is reported and known as flipped ambiguity [25][26][27]. In case the neighborhood of a node are located in such way that they are approximately on a same line then the estimated position may be in the flipped location with respect to the line. Fig. 3 shows a deployment with 30% of anchors with bigger error not only due to the less number of anchor nodes but also due to the aforementioned flipped ambiguity. The other source of error is the absence of anchor in a region due to the non-uniform distribution of the anchor nodes. The flipped neighborhood indicated in the Fig. 3 shows the uneven distribution of the anchor in the specified region. In some cases the whole neighborhood is flipped and contributes to upsurge of error. Correcting the flipped ambiguity in the CE localization technique necessitates further research and we have intention to contribute to this area in our future work.

Fig. 4 shows the error in successive rounds. The error decays exponentially. Therefore with a small number of iterations the algorithm converges to its minima. Though the figure demonstrates a single event of error in rounds we observe many instances and almost always this is the case where the convergence is quick and efficient. This is an important criterion of selecting an optimization algorithm. A small number of rounds in convergence demonstrate algorithm efficiency in term of its cost. Fig. 5 displays the estimated locations of a specific node in rounds alternatively the searching path of that particular run. Both Fig. 4 and Fig. 5 conform that the search converges to the minima with exponentially decayed cost.

In order to evaluate the performance of our proposed CE algorithm, we compare it against the two well-known localization algorithms, namely, DV-Hop algorithm [15] and Simulated Annealing (SA)-based algorithm [23].

SA algorithm takes much more iterations to converge compared to CE. On the other hand per iteration of CE takes longer time than that of SA. Therefore to make a fair comparison Fig. 6 shows the error performance of CE and SA algorithm with respect to time thereby depicts the core algorithmic efficiency of CE over SA.

Fig. 7 shows error performance of individual nodes with 30% of anchor nodes. Intuitively the big spikes in the CE method are due to the flipped nodes and can be eliminated by incorporating appropriate measure. Fig. 8 shows algorithm error performance compared with different percentages of the anchor deployments. Here each error point is calculated by averaging 10 measurements. Both of the figures reveal that DV-Hop provides a poor performance compare to the other two. When there are less number of anchors the error becomes more and more is a common phenomenon for all the three cases which is quite expected though DV-Hop has the worst increasing rate of error with decreasing percentage of anchors. Especially the performance becomes too poor when the percentage of anchor node is small. On the other hand SA approach provides the best error performance but with a cost of slow algorithmic convergence. Fig. 9 demonstrates the poor efficiency of SA algorithm in terms of its algorithmic runtime.

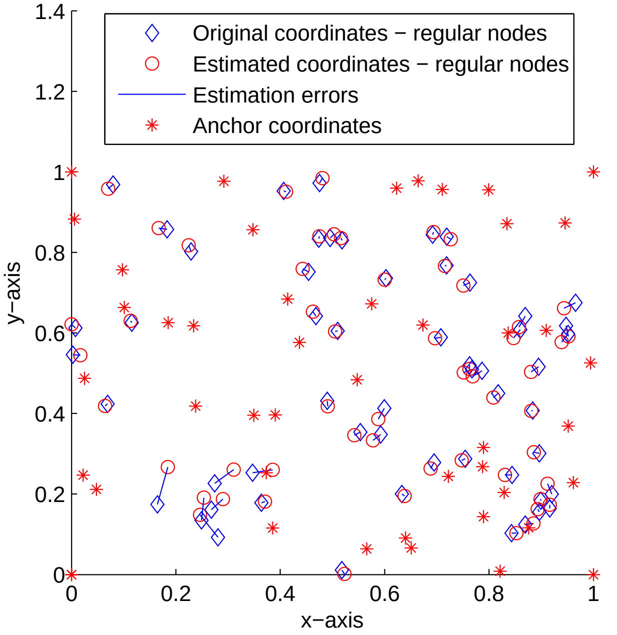

Fig. 2. Node locations in the network where the anchors are 45% of the total nodes. Distances are normalized into the range 0.0 to 1.0.

Therefore in case of a large mobile sensor application SA approach of localization can never be justifiable because of its higher processing cost. Alternatively the proposed algorithm becomes a suitable approach of localization with a reasonably low processing cost with a little sacrifice of localization accuracy. One simplest and straight forward way to determine the required number of rounds in algorithm is to track the error in successive rounds. The algorithm exit from the iterative loop if the current performance compare to the previous performance does not improve more than a predefined threshold. Therefore the runtime is calculated by time stamping the n^{th} round when the $(n+1)^{th}$ round cannot bring the error further down. 10 measurements are taken to get the runtime average for each deployment.

IV. CONCLUSIONS AND FUTURE WORKS

A novel cross-entropy-based localization algorithm is devised in the context of wireless sensor networks. The algorithm attempts to estimate the locations of the nodes in the networks centrally from the distance measured based on transmit-receive signal strengths. Error introduced by the unreliable wireless communications is minimized by CE based optimization technique. Simulation results show that the algorithm can estimate the location coordinate of sensor nodes with reasonably good accuracy with low computational costs. Mobile sensor network with large number of nodes can be benefited by this computationally efficient localization technique.

The cost function of CE takes equal weights for all the

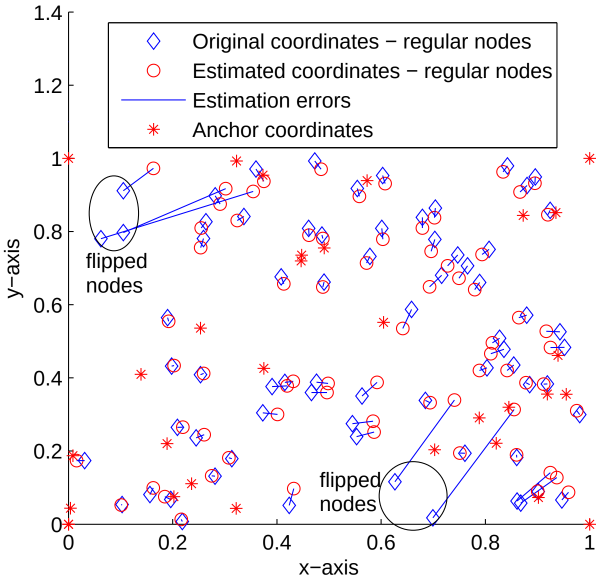

Fig. 3. Node locations in the network where the anchors are 30% of the total nodes. Distances are normalized into the range 0.0 to 1.0.

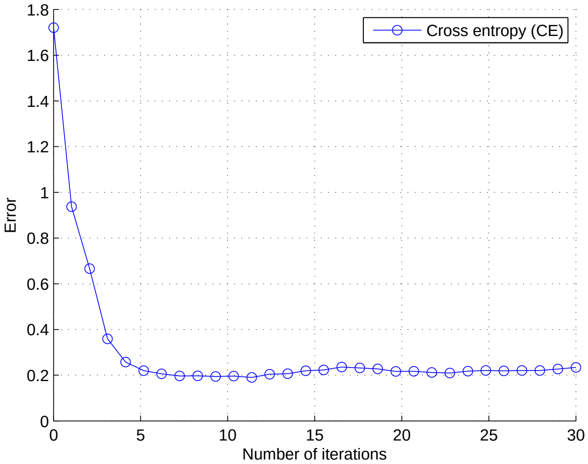

Fig. 4. Numbers of errors in rounds.

neighbor distances in the neighborhood. In practice some neighbor information is more reliable than the other [28]. Therefore a possible future improvement of the algorithmic cost function is to incorporate weights base on the reliability of the particular neighbor. We also intend to contribute to the area of flipped ambiguity problem in the CE localization approach one common drawback of the error minimizing technique based estimation algorithm for localization.

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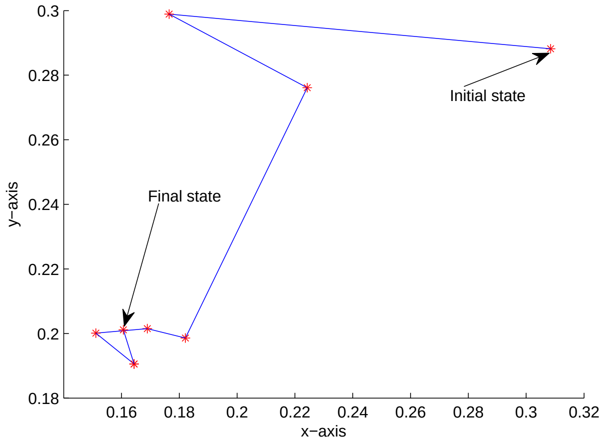

Fig. 5. Estimated location of a specific sample node in rounds.

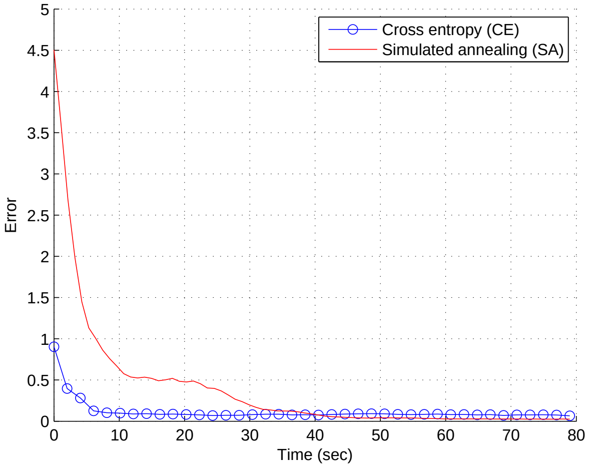

Fig. 6. Error over algorithmic runtime.

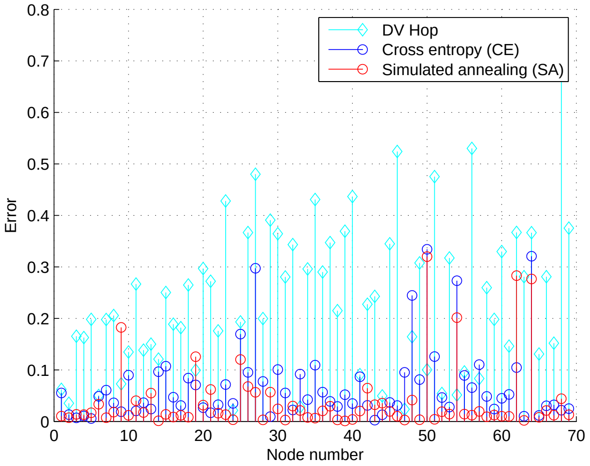

Fig. 7. Error in each node.

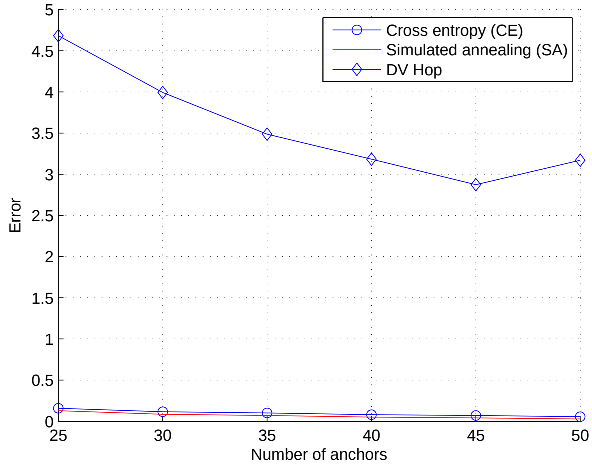

Fig. 8. Average error.

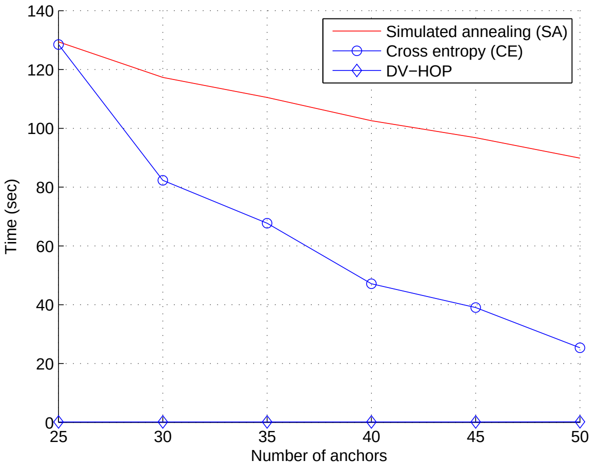

Fig. 9. Simulation runtime.

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